HEAT TRANSFER FROM ELLIPSOIDAL PARTICLES

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The paper gives a numerical solution of the steady convective heat transfer from circular ellipsoid shaped particles in a confined laminar region. The effect of eccentricity, the Peclet number and the restriction of the space domain on the magnitude of the Nusselt number is shown on numerical results. The paper examines also the effect of variation of parameters on distribution of intensity of heat transfer in the vicinity of the particle and on the thickness of the boundary layer. A comparison of theoretical results with experimental ones published to date shows a good agreement.

The problem of convective heat transfer from particles is often encountered in chemical engineering practice. The research effort in this field has been directed predominantly to experimental investigation of the effect of conditions on intensity of heat transfer. The hydrodynamic and thermal conditions at convective heat transfer in a bed of particles are too complex to be treated mathematically. Accordingly, the theoretical papers concentrate on solution of a substantially simplified problem. The most often investigated situation is that of convective heat transfer from a spherical particle to an infinite laminar medium[1]. Both the experimental and the theoretical studies show that the disturbances in the thermal and flow fields induced by the presence of the particle diminish with the distance from the particle.

Lamb[2] solved the Navier–Stokes equation for low values of the Reynolds number and assumed a constant velocity on the spherical surface characterizing the restriction of the space domain avialable for one particle. Thus he approximated simultaneous motion of several particles without mutual contact by a set of individually moving particles each in its own delimited region. Similar approach was used by Pffefer and Happel[3] to solve the intensity of heat transfer from a spherical particle to a laminar medium. In the Fourier equation, characterizing the convective transfer, they used Lamb's solution for the flow field. In addition, they assumed a constant temperature on the surface delimiting the space available for the particle. Their numerical results are comparable with experimental ones for $\mathrm{Pe} < 1$. Another paper[4] gives a solution of the same problem also for $\mathrm{Pe} \leqq 100$. Theoretical solution of convective transfer from the surface of non-spherical particles in an infinite laminar region is provided by Brenner[5]. Solution of the Navier–Stokes equation for the laminar flow past the circular ellipsoid shaped particles in an infinite fluid is given by Happel and Brenner[6].

In the present paper we shall make use of some results of Happel and Brenner[6] and derive the flow field in the neighbourhood of circular ellipsoidal particles, assuming a constant velocity on the surface of a circular ellipsoid of the same eccentricity with collinear axes. If moreover the temperature on this surface is assumed constant, we can also calculate intensity of heat transfer in the confined laminar region.

EQUATION OF CONVECTIVE HEAT TRANSFER FROM CIRCULAR ELLIPSOID IN A RESTRICTED LAMINAR REGION

Mathematical model of the process of convective heat transfer is basically an expression of the Fourier law

$$\mathbf{v} \,.\, \operatorname{grad} T = a_{\mathrm{tv}} \nabla^2 T \tag{1}$$

The dimensionless temperature $T = 1$ at the surface of the particle and $\nabla^2 T = 0$ on the boundary of the restricted region.

Oblate Ellipsoids

Since our particle is a circular ellipsoid it is convenient to transform Eq. (*1*) into elliptic coordinates. Elliptic coordinates, however, are different for oblate and prolate ellipsoids. Accordingly, the transformation of Eq. (*1*) will be derived separately for both types of coordinates.

A detailed description of these coordinates, including instructive figures and an outline of the transformation, is given by Happel and Brenner[6]. Introducing $u = \sinh \xi$, $x = \cos \eta$ we may write[6] for the stream function ψ

$$\psi = (1 - x^2) \left\{-\tfrac{1}{2}c_1 u + \tfrac{1}{2}c_2[u - (u^2 + 1) \operatorname{arccotg} u] + c_3(u^2 + 1)\right\}, \tag{2}$$

where c_1, c_2, c_3 are integration constants. Equation of an ellipsoid in these coordinates takes the form $u = u_0 = \text{const}$. Consider the equation of an envelope of the restricted region $u = u_1$. For u_0 and u_1 we then have relation $u_0 = du_1$, where d is a quantity characterizing the restriction ($d < 1$). If d decreases, the envelope of the restricted region departs from the surface of the particle. For $d = 0$, the space becomes unrestricted. Happel and Brenner[6] determine the integration constants for this case only. They do not introduce the envelope of restriction.

The integration constants c_1, c_2, c_3 are determined from the boundary conditions:

$$\psi = 0 \quad \text{and} \quad \partial\psi/\partial u = 0 \quad \text{for} \quad u = u_0$$

$$\psi = \tfrac{1}{2}Uc^2(u^2 + 1)(1 - x^2) \quad \text{for} \quad u = u_1$$

On substituting into Eq. (*2*) we get:

$$\begin{gathered} -\tfrac{1}{2}u_0 c_1 + \tfrac{1}{2}[u_0 - (u_0^2 + 1) \operatorname{arccotg} u_0]\, c_2 + (u_0^2 + 1)\, c_3 = 0\,, \\ -\tfrac{1}{2}c_1 + (1 - u_0 \operatorname{arccotg} u_0)\, c_2 + 2u_0 c_3 = 0\,, \\ -\tfrac{1}{2}u_1 c_1 + \tfrac{1}{2}[u_1 - (u_1^2 + 1) \operatorname{arccotg} u_1]\, c_2 + (u_1^2 + 1)\, c_3 = \tfrac{1}{2}Uc^2(u_1^2 + 1)\,. \end{gathered} \tag{3}$$

For the eccentricity we have that $c = (a^2 - b^2)^{1/2}$. If the eccentricity of the ellipse (a cut through a given particle) is known and if the magnitude of the major axis is assumed to be a_0 we may write $u_0 = (a_0^2 - c^2)^{1/2}$. For a given restriction d we then may write $u_1 = u_0/d$. Thus if the eccentricity of a given ellipsoidal particle, the magnitude of its major axis a_0, and the restriction d are known, the coefficients c_1, c_2, c_3 of the system (*3*) are known too. System (*3*) is then a set of three linear equations in three variables. Since the right hand sides of the system (*3*) are directly proportional to the product Uc^2, the roots will be scaled by this product too. Substituting unity for this product at solution, we obtain roots c'_1, c'_2, c'_3 and for c_1, c_2, c_3 we have $c_1 = c^2 Uc'_1$ ... *etc*. For the ellipsoid forming the boundary follows, on condition that the eccentricity is the same and $u_1 = u_0/d$, that

$$a_1 = [a_0^2/d^2 - c^2(1/c^2 - 1)]^{1/2}\,, \quad b_1 = b_0/d$$

From the relations given by Happel and Brenner[6] in the already cited paper we determine the ellipsoidal components of velocity

$$v_\eta = \frac{Uc^2(1 - x^2)\,(-c'_1/2 + c'_2\,[1 - u\,\mathrm{arccotg}\,u] + 2c'_3 u)}{c^2[(u^2 + x^2)\,(1 - x^2)]^{1/2}}\,,$$

$$v_\xi = \frac{Uc^2\,2x\,[-(c'_1/2)\,u + (c'_2/2)\,[u - (u^2 + 1)\,\mathrm{arccotg}\,u] + c'_3(u^2 + 1)]}{c^2[(u^2 + x^2)\,(1 - x^2)]^{1/2}}\,.$$

After transformation, substitution for v_η and v_ξ and some arrangements, Eq. (*1*) takes the form

$$\frac{\mathrm{Pe}}{2}\,c'\left\{\frac{2x\left\{-(c'_1/2)\,u + (c'_2/2)\left[u - (u^2 + 1)\,\mathrm{arccotg}\,u\right] + c'_3(u^2 + 1)\right\}}{(1 + u)^{1/2}}\,\frac{\partial T}{\partial \xi} + \right.$$
$$\left. + \frac{(1 - x^2)\left[-(c'_1/2) + (c'_2/2)\,(1 - u\,\mathrm{arccotg}\,u) + 2c'_3 u\right]}{(1 - x^2)^{1/2}}\,\frac{\partial T}{\partial \eta}\right\} =$$
$$= \frac{1}{(1 + u^2)^{1/2}}\,\frac{\partial}{\partial \xi}\left[(1 + u^2)^{1/2}\,\frac{\partial T}{\partial \xi}\right] + \frac{1}{(1 - x^2)^{1/2}}\,\frac{\partial}{\partial \eta}\,(1 - x^2)^{1/2}\,\frac{\partial T}{\partial \eta}\,, \qquad (4)$$

where c', the dimensionless eccentricity, equals c/a.

The domains of variables are $0 \leqq \eta \leqq \pi$, arsinh $u_0 \leqq \xi \leqq$ arcsinh u_1. The boundary conditions are $T(u_0), = 0$, $T(u_1) = 1$. It is convenient to rearrange Eq. (*4*) by substituting $x = \cos \eta$ and consistently in derivatives too:

$$\partial/\partial\eta = -(1 - x^2)^{1/2}\,\partial/\partial x\,. \qquad (5)$$

Now we shall transform the coordinate ξ. For the application of the finite difference method of solution it is convenient to implement the inverse transformation $y = q_1/u + q_2$ so as to get $u = u_0$ at $y = 0$ and $u_1 = u_0/d$ at $y = 1$. By these substitutions the domain of the solution transforms into a rectangle

$$-1 \leqq x \leqq 1\,; \quad 0 \leqq y \leqq 1\,. \qquad (6)$$

Besides, inverse transformation in y direction provides that major portion of the interval $\langle 0 - 1 \rangle$ for y is crowded near the surface of the particle where major changes in temperature occur. The pertinent relations for y have the following form:

$$y = \frac{u_0}{(1 - d)\, u} - \frac{d}{1 - d}, \quad u = \sinh \xi ,$$

$$\partial/\partial\xi = \frac{u_0}{(1 - d)\, u^2} (1 + u^2)^{1/2}\, \partial/\partial y . \tag{7}$$

Performing the transformation of the derivatives and after some arrangement we get:

$$c' \operatorname{Pe}/2 \Big[2x\{-c_1' u/2 + (c_2'/2)\,[u - (u^2 + 1) \operatorname{arccotg} u] +$$

$$+ c_3'(u^2 + 1)\} \frac{u_0}{(1 - d)\, u^2} \frac{\partial T}{\partial y} + (1 + x^2)[c_1'/2 + c_2'(1 - u \operatorname{arccotg} u) + 2c_3' u] \frac{\partial T}{\partial x} \Big] +$$

$$+ \frac{u_0}{(1 - d)^2\, u^4} (1 + u^2) \frac{\partial^2 T}{\partial y^2} + \frac{2}{(1 - d)\, u^3} \frac{\partial T}{\partial y} + (1 - x^2) \frac{\partial^2 T}{\partial x^2} - 2x \frac{\partial T}{\partial x} = 0 . \tag{8}$$

Thus we arrive at partial differential equation of the second order and elliptic within the domain of interest. The coefficients of the derivatives are functions of x, y. The domain of solution is given as $-1 \leqq x \leqq 1$, $\pm 0 \leqq y \leqq 1$. The boundary conditions are given as $T(y = 0) = 0$. $T(y = 1) = 1$. For $x = \pm 1$ we request that $\partial T/\partial x = 0$.

Prolate Ellipsoids

Introducing

$$x = \cos \eta , \quad v = \cosh \xi ,$$

the stream function ψ may be written as:

$$\psi = (1 - x^2) \{-c_1 v/2 + c_2/2[v - (v^2 - 1) \operatorname{arccotgh} v] + c_3(1 - v^2)\} . \tag{9}$$

c_1, c_2, c_3 are integration constants to be determined from the boundary conditions. Equation of an ellipsoid in these coordinates takes the form $v = v_0 = \text{const}$. The equation of the envelope of the restricted region is taken as $v = v_1$. Between v_0 and v_1 we have that $v_0 = dv_1$, where d is a quantity characterizing restriction ($d < 1$). The lateral axis of a longitudinal axial cut is designated again as a_0. This time, however, a_0 is the minor axis. The eccentricity is $c = (b_0^2 - a_0^2)^{1/2}$. Following quantities are given as parameters of the solution: the eccentricity c, the length of axis a_0 and the restriction d. The value of v_0 is obtained as a ratio $v_0 = b_0/c = (c^2 + a_0^2)^{1/2}/c$. The values of c_1, c_2, c_3 are determined from the boundary conditions:

$$\psi = 0 \quad \text{for} \quad v = v_0 ,$$

$$\partial\psi/\partial v = 0 \quad \text{for} \quad v = v_0 ,$$

$$\psi = \tfrac{1}{2}Uc^2(v^2 - 1)(1 - x^2) \quad \text{for} \quad v = v_1 .$$

On substitution into Eq. (9) we get:

$$-\frac{c_1}{2} v_0 + \frac{c_2}{2} [v_0 - (v_0^2 - 1) \operatorname{arccotgh} v_0] + c_3(1 - v_0^2) = 0 ,$$

$$-\frac{c_1}{2} + c_2(1 - v_0 \operatorname{arccotgh} v_0) - 2c_3 v_0 = 0 , \tag{10}$$

$$-\frac{c_1}{2} + \frac{c_2}{2} [v_1 - (v_1^2 - 1) \operatorname{arccotgh} v_1] + c_3(1 - v_1) = \tfrac{1}{2}c^2 U(v_1^2 - 1) .$$

For given v_0, v_1 the solution is again scaled by expression $c^2 U$. Thus we substitute unity for $c^2 U$, determine c_1', c_2', c_3' and write $c_1 = c^2 Uc_1'$, $c_2 = c^2 Uc_2'$, $c_3 = c^2 Uc_3'$. The shape of the restricting surface may be obtained from conditions of equal eccentricity and $v_1 = v_0/d$. The first condition leads to $b_1 = b_0/d$; the second leads then to

$$a_1 = \left(\frac{a_0^2}{d^2} + c^2\left(\frac{1}{d^2} - 1\right)^2\right)^{1/2} .$$

The components of velocity can be determined similarly as in the case of oblate ellipsoids:

$$v_\xi = \frac{Uc^2\, 2x \left\{-\frac{c_1'}{2} + \frac{c_2'}{2} [v - (v^2 - 1) \operatorname{arccotg} v] + c_3'(1 - v^2)\right\}}{c^2[(x^2 - v^2)(v^2 - 1)]^{1/2}} ,$$

$$v_\eta = \frac{Uc^2(1 - x^2)\left[-\frac{c_1'}{2} + c_2'(1 - \operatorname{arccotgh} v)\right] - 2c_3' v}{c^2[(x^2 - v^2)(1 - x^2)]^{1/2}} \tag{11}$$

After transformation and substitution for v_ξ and v_η, Eq. (1) takes the form

$$\frac{\mathrm{Pe}}{2} c' \left\{ \frac{2x - (c_1'/2) + (c_2'/2)[v - (v^2 - 1) \operatorname{arccotgh} v] + c_3'(1 - v^2)}{(v^2 - 1)^{1/2}} \frac{\partial T}{\partial \xi} + \right.$$

$$\left. + \frac{(1 - x^2)[-(c_1'/2) + c_2'(1 - v \operatorname{arccotgh} v) - 2c_3' v]}{(1 - x^2)^{1/2}} \frac{\partial T}{\partial \xi} \right\} = \tag{12}$$

$$= \frac{1}{(v^2 - 1)^{1/2}} \frac{\partial}{\partial \xi} (v^2 - 1)^{1/2} \frac{\partial T}{\partial \xi} + \frac{1}{(1 - x^2)^{1/2}} \frac{\partial}{\partial \eta} (1 - x^2)^{1/2} \frac{\partial T}{\partial \eta} .$$

Pe is again the Peclet number $\mathrm{Pe} = (2Ua_0)/a_{(v)}$, c' is the dimensionless eccentricity defined as $c' = c/a_0$. The domains of individual variables are:

$$0 \leqq \eta \leqq \pi\,; \quad \operatorname{arccosh} v_0 \leqq \xi \leqq \operatorname{arccosh} v_1$$

and the boundary conditions:

$$T(v_0) = 0\,; \quad T(v_1) = 1\,.$$

Similar transformation as in Eq. (*6*) is performed with Eq. (*12*):

$$x = \cos \eta\,, \quad \frac{\partial}{\partial \eta} = -(1 - x^2)^{1/2} \frac{\partial}{\partial x}\,,$$

$$y = q_1/v + q_2\,,$$

where q_1, q_2 are such so as $y(v)_1 = 1$ and $y(v_0) = 0$. These conditions lead to the following substitution

$$y = \frac{1}{(1 - d)\,v} - \frac{d}{1 - d}\,; \quad v = \cosh \xi\,;$$

$$\frac{\partial}{\partial \xi} = \frac{v_0}{(1 - d)\,v^2} (v^2 - 1)^{1/2} \frac{\partial}{\partial y}\,.$$

Having substituted for the derivatives in Eq. (*12*) and after some arrangement we obtain relation:

$$c' \frac{\mathrm{Pe}}{2} \left[- \frac{2v_0}{(1 - d)^2}\, x \left\{ \frac{c_1'}{2}\, v + \left(\frac{c_2'}{2} \right) [v - (v^2 - 1 \operatorname{arccotgh} v] + c_3'(1 - v^2) \right\} \frac{\partial T}{\partial y} + \right.$$
$$\left. + (1 - x^2) \left[- \frac{c_1'}{2} + c'(1\text{-}\operatorname{arccotgh} v) - 2c_3' v \right] \frac{\partial T}{\partial x} \right] + \qquad (13)$$
$$+ \frac{v_1^2(v^2 - 1)}{(1 - d)^2\, v^4} \frac{\partial^2 T}{\partial y^2} - \frac{2v_0}{(1 - d)\, v^3} \frac{\partial T}{\partial y} + (1 - x^2) \frac{\partial T}{\partial x^2} - 2x \frac{\partial T}{\partial x} = 0$$

$$-1 \leqq x \leqq 1\,; \quad 0 \leqq y \leqq 1\,; \quad T(0) = 0; \quad T(1) = 1\,, \quad \frac{\partial T}{\partial x} = 0 \text{ for } x = \pm 1.$$

c', d and Pe in this equation are given parameters. c_1', c_2', c_3' are numerical values of the roots of the set of linear equations (*10*) $v = v_0/(y(1 - d) + d)$.

Eq. (*13*) is thus also a partial differential equation of the second order of the elliptic type. The domain of the solution and the boundary conditions remain the same as for Eq. (*8*) pertaining to oblate ellipsoids.

Eqs (*8*) and (*13*) are formally of the same type. Both are solved by the finite difference method using a program presented in the preceding paper[7]. The finite difference method enables the temperature in preset grid points to be determined. Numerical derivation, integration and interpolation permit then to find a series of additional properties of the problem for selected parameters Pe, d and b/a. Several computational runs for different values of parameters and evaluation of the results enables us to examine the dependence of evaluated quantities on varying parameters.

Numerical interpolation gives the pattern of isotherms in the neighborhood of the particle. If the local thickness of the boundary layer is defined as the distance of the $T = 0{\cdot}01$ isotherm, then the former may also be determined by numerical interpolation. By numerical derivation we determine temperature gradients in the neighborhood of the particle. The value of the Nusselt number is determined from the relation $\mathrm{Nu} = -1/S \int \mathrm{d}\mathbf{S}\ \mathrm{grad}\ T$ by numerical integration.

RESULTS

Individual calculated quantities were changing with variation of parameters. The course of the dependence of Nu on Pe at different restrictions and the ratio of the axes $b/a = 0{\cdot}8$ is shown in Fig. 1. The figure also shows that in an infinite region the value of the Nusselt number for oblate ellipsoids and low values of the Peclet number is smaller than 2. The value of Nu increases in all cases with increasing Pe. The increase is slower for smaller values, faster for higher ones. With gradually increasing restriction d the value of the Nusselt number increases, and, simultaneously, the steeper part of the Nu = f(Pe) curve moves toward higher values of Pe. Fig. 2 shows the dependence of the Nusselt number on Pe for ellipsoidal particles of different eccentricity at zero restriction. Curves *1* through *4* give the dependence of Nu on Pe for oblate ellipsoids and the ratio of axes 0·99, 0·8, 0·4 and 0·1. Curves *5* and *6* correspond to the dependences for prolate ellipsoids. Curve *5* corresponds to ellipsoidal particles with the ratio of longitudinal to lateral axis 0·8; curve *6* for that ratio equalling 0·6. The plot of the relation between Nu and Pe for prolate ellipsoids with the ratio of axes 0·99 is identical with that for oblate ellipsoids and the same ratio of axes. It is observed that for low values of Pe the Nusselt number increases with increasing longitudinal axis.

A function Nu = f(Pe, c', d) was searched for by a linear regression with the following relations as the result:

$$\mathrm{Nu} = 2{\cdot}54\ \mathrm{Pe}^{0.10}\ c'^{0.0830}(1 - d)^{-1.45}\,, \quad \text{eventually}$$

$$\mathrm{Nu} = 3{\cdot}25\ \mathrm{Pe}^{0.091}\ c'^{0.050}(1 - d^3)^{-4.54} \quad \text{for } \textit{oblate ellipsoids};$$

$$\mathrm{Nu} = 4{\cdot}36\ \mathrm{Pe}^{0.278}\ c'^{0.080}(1 - d)^{-0.478}\,, \quad \text{eventually}$$

$$\mathrm{Nu} = 4{\cdot}68\ \mathrm{Pe}^{0.278}\ c'^{0.080}(1 - d^3)^{-1.59} \quad \text{for } \textit{prolate ellipsoids.}$$

The $(1 - d^3)$ quantity characterizes the void space belonging to one particle. From the relations it is apparent that Nu markedly increases with increasing restriction. The increase is somewhat less pronounced with varying value of Pe. The smallest change then occurs with changing eccentricity.

Interesting results yields also investigation of the distribution of intensity of heat transfer along the surface of the particle. A good insight is provided by examining the ratio, p, of intensity of heat transfer on the trailing edge of the particle to that on the leading edge. Following relations were obtained by multiple linear regression $p = 1{\cdot}28\ \mathrm{Pe}^{-0.013}\ c'^{0.17}(1 - d)^{0.33}$, eventually

$$p = 1{\cdot}18\ \mathrm{Pe}^{-0.013}\ c'^{0.17}\ (1 - d^3)^{0.68} \quad \text{for } \textit{oblate ellipsoids;}$$

$$p = 0{\cdot}477\ \mathrm{Pe}^{-0.33}\ c'^{0.082}(1 - d)^{-0.50}\,, \quad \text{eventually}$$

$$p = 0{\cdot}535\ \mathrm{Pe}^{-0.33}\ c'^{0.082}(1 - d^3)^{-1.21} \quad \text{for } \textit{prolate ellipsoids.}$$

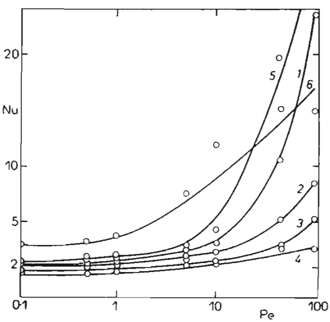

FIG. 1

Dependence of Nu on Pe for Ellipsoidal Particles and the Ratio $b/a = 0{\cdot}8$

1 Restriction d 0; *2* d 0·2; *3* d 0·4; *4* d 0·6.

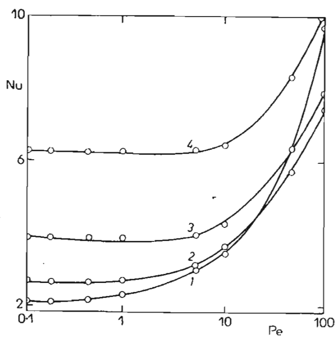

FIG. 2

Dependence of Nu on Pe for Ellipsoidal Particles at $d = 0$ and Different Ratio of Axes b/a

1 $b/a = 0{\cdot}99$; *2* $b/a = 0{\cdot}8$; *3* $b/a = 0{\cdot}4$; *4* $b/a = 0{\cdot}1$; *5* $b/a = 1{\cdot}25$; *6* $b/a = 2{\cdot}5$.

From the magnitude of the coefficients of the regression function it may be judged that the intensity of heat transfer on the trailing edge, in comparison with that on the leading edge, decreases with Pe. A faster decrease occurs for prolate ellipsoid shaped particles. The ratio p depends directly on the dimensionless eccentricity. For more oblate particles the relative difference in intensity of heat transfer is smaller. The same dependence, though somewhat less conspicuous, is valid for prolate ellipsoids.

With increasing restriction the relative difference in intensity of the fluxes increases (p decreases) for oblate ellipsoid shaped particles. For prolate ones, p increases with increasing restriction, *i.e.* the relative difference in intensity of the fluxes decreases.

Information about the extent of the thermal field, *i.e.* the part of the space sustaining substantial temperature differences, may be obtained from the thickness of the boundary layer. If the latter is defined as the average distance of the $T = 0{\cdot}01$ isotherm from the surface of the particle[4] then it is possible to determine this value by interpolation. The dependence of $\bar{\delta}$ on selected parameters can also be examined in graphical form.

Figs 3 and 4 show the dependence of the average thickness of the boundary layer $\bar{\delta}$ on the Peclet number. The restriction is a parameter in Fig. 3 and $b/a = 0{\cdot}8$. Fig. 4 gives that dependence for different ellipsoids and $d = 0$. Only two curves are drawn in Fig. 4 since it was found that at constant restriction individual curves differ only very little. The curves suggest that the thickness of the boundary layer decreases

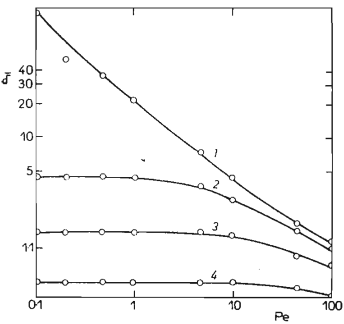

FIG. 3

Dependence of $\bar{\delta}$ on Pe for Ellipsoidal Particles at the Ratio of Axes $b/a = 0{\cdot}8$

1 Restriction d 0; *2* d 0·2; *3* d 0·4; *4* d 0·6.

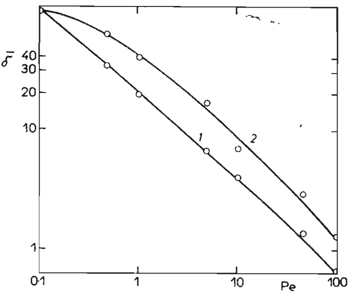

FIG. 4

Dependence of $\bar{\delta}$ on Pe for Ellipsoidal Particles at $d = 0$ and Different Ratio of Axes b/a

1 $b/a = 0{\cdot}99$; *2* $b/a = 1{\cdot}25$.

both with increasing Pe and restriction. The differences in the thickness of the boundary layer at different restrictions diminish with increasing value of Pe.

It may be concluded that the finite difference method permits by taking the double of the increment to determine the error of the temperature in individual grid points. Thus determined errors were always smaller than 1‰. The errors of Nu were even smaller.

Comparison with the literature. The results for very small eccentricity, *i.e.* the ellipsoids with the ratio of axes approaching unity may be compared with the results for spherical particles. Heat transfer from the surface of a solid spherical particle in a confined laminar region has been solved in paper[4]. Our results for the axes ratio equal 0·999 are comparable with those published in the mentioned paper. Solution of heat (and mass) transfer from non-spherical particles in an infinite laminar region is given by Brenner[5]. His results are applicable for $Pe < 1$. For the Nusselt number the author gives the relation $Nu = Nu_0(1 + \frac{1}{8} Nu_0 Pe)$. Nu_0 here stands for Nu at $Pe = 0$.

Comparing our results with Nu calculated from the last relation[5] for $Pe < 1$ we obtain deviations of the order 8% for particles with the axis ratio 0·999; for particles with a smaller value of this ratio (greater eccentricity) the deviations decrease. For particles with the axis ratio 0·1 the deviations amount to 2%. These results, however, cannot be applied to particles in a confined region. In such a case we obtain for lower value of Pe slightly changing values of Nu. With increasing value of the restriction the range for Pe within which Nu changes only slightly widens.

LIST OF SYMBOLS

a_0 length of lateral axis of the principal longitudinal cut through particle
a_1 length of lateral axis of the principal longitudinal cut through restricting boundary
a_{tv} thermal diffusivity
b_0 length of longitudinal axis of the principal longitudinal cut through particle
b_1 length of longitudinal axis of the principal cut through restricting boundary
$c = (a_0^2 - b_0^2)^{1/2}$ eccentricity for oblate ellipsoids
$c = (b_0^2 - a_0^2)^{1/2}$ eccentricity for prolate ellipsoids
$c' = c/a$ dimensionless eccentricity
d restriction
$Nu = (2\alpha a_0)/\lambda$ Nusselt number
$Pe = (2Ua_0)/a_{tv}$ Peclet number
p ratio of intensity of heat transfer on the leading edge to that on the trailing edge
S surface area of particle
t temperature
t_0 temperature on surface of particle
t_∞ temperature on restricting boundary
$T = (t - t_\infty)/(t_0 - t_\infty)$ dimensionless temperature
$u = \sinh \xi$ transformation for oblate ellipsoids
$u_0 = u$ equation of surface for circular oblate ellipsoid shaped particle
$u_1 = u$ equation of restricting surface for oblate ellipsoid particle
U velocity on restricting boundary
$v = \cosh \xi$ transformation for prolate ellipsoids
$v_0 = v$ equation of surface for circular prolate ellipsoid shaped particle
$v_1 = v$ equation of restricting surface for prolate ellipsoids

v	velocity vector
$x = \cos \eta$	transformation
$y = \dfrac{u_0}{(1-d)\,u} - \dfrac{d}{1-d}$	transformation for prolate ellipsoids
α	heat transfer coefficient
$\bar{\delta}$	boundary layer thickness
ξ	elliptic coordinate
λ	thermal conductivity of fluid
ψ	stream function

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